M circles and Nichols Chart

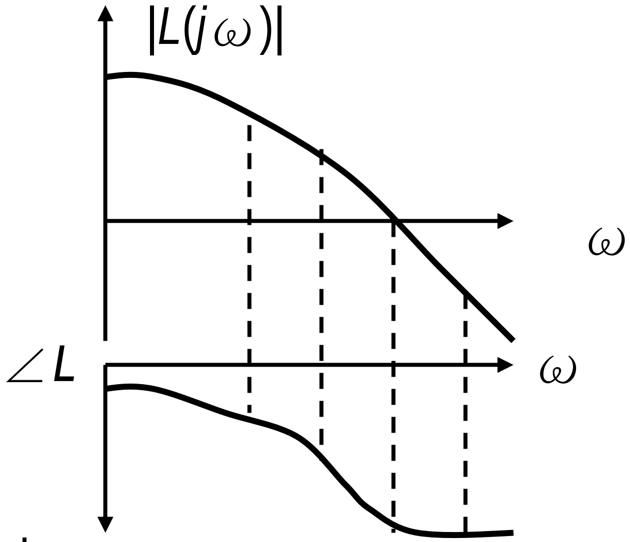
Consider a candidate design of a loop transfer function $L(j\omega)$ shown on the RHS.

The equality

$$T(j\omega)=\frac{L(j\omega)}{1+L(j\omega)}$$

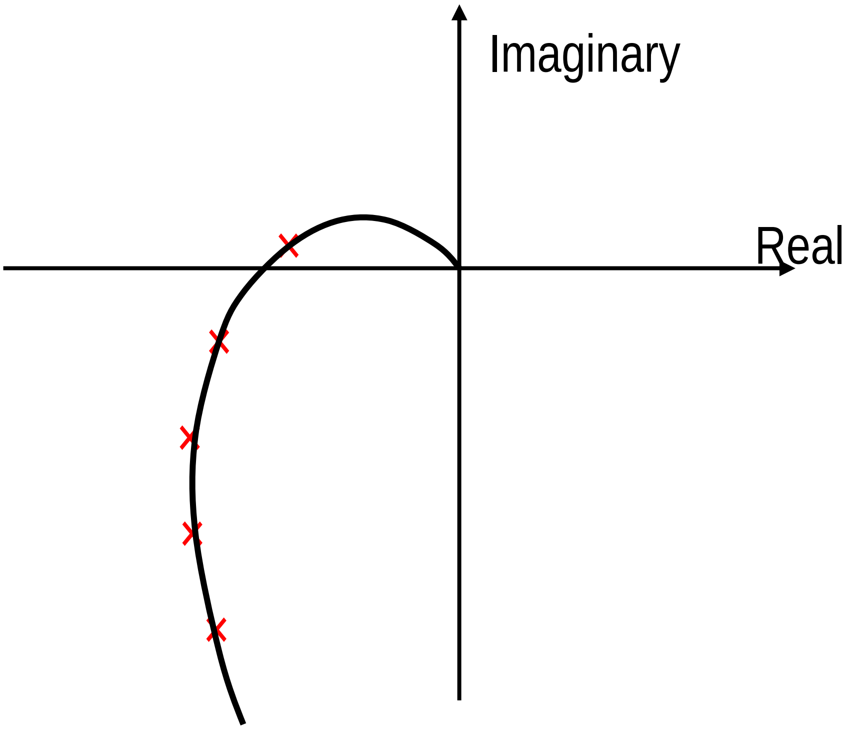
holds true freq. by freq.

for all $\omega\in[0, \infty)$



Hence, in theory, we can precisely Evaluate $T(j\omega)$ from $L(j\omega)$ in the manner of frequency point by frequency point.

Alternatively, the Bode plot of $L(j\omega)$ can also be show on the complex plane to form its Nyquist plot



§M circles (constant magnitude of T)

In order to precisely evaluate $|T(j\omega)|$ from the Nyquist plot of $L(j\omega)$, a tool called M circle is developed as followed.

Let $L(j\omega)=X+jY$, where *X* is the real and *Y* the imaginary part. Then

$$|T(j\omega)|=M=\frac{|X(j\omega)+jY(j\omega)|}{|1+X(j\omega)+jY(j\omega)|},$$

and it follows

$$M(j\omega)^2=\frac{X(j\omega)^2+Y(j\omega)^2}{\left(1+X(j\omega)\right)^2+Y(j\omega)^2}$$

Rearranging the above equation, it gives

$$X^2(1-M^2)-2M^2X-M^2+(1-M^2)Y^2=0 \qquad (\#1)$$

(i) If M = 1, it follows from (#1) that $X=\frac{-1}{2}$, Y= anything

hence all (X, Y) pair that correspond to M=1 is a vertical line on the complex plane.

(ii) If M $\neq$ 1, (#1) reduces to

$$X^2+\frac{2M^2}{M^2-1}X+\frac{M^2}{M^2-1}+Y^2=0$$

Adding $M^2/\left(M^2-1\right)^2$ to both sides of the above eq., it gives

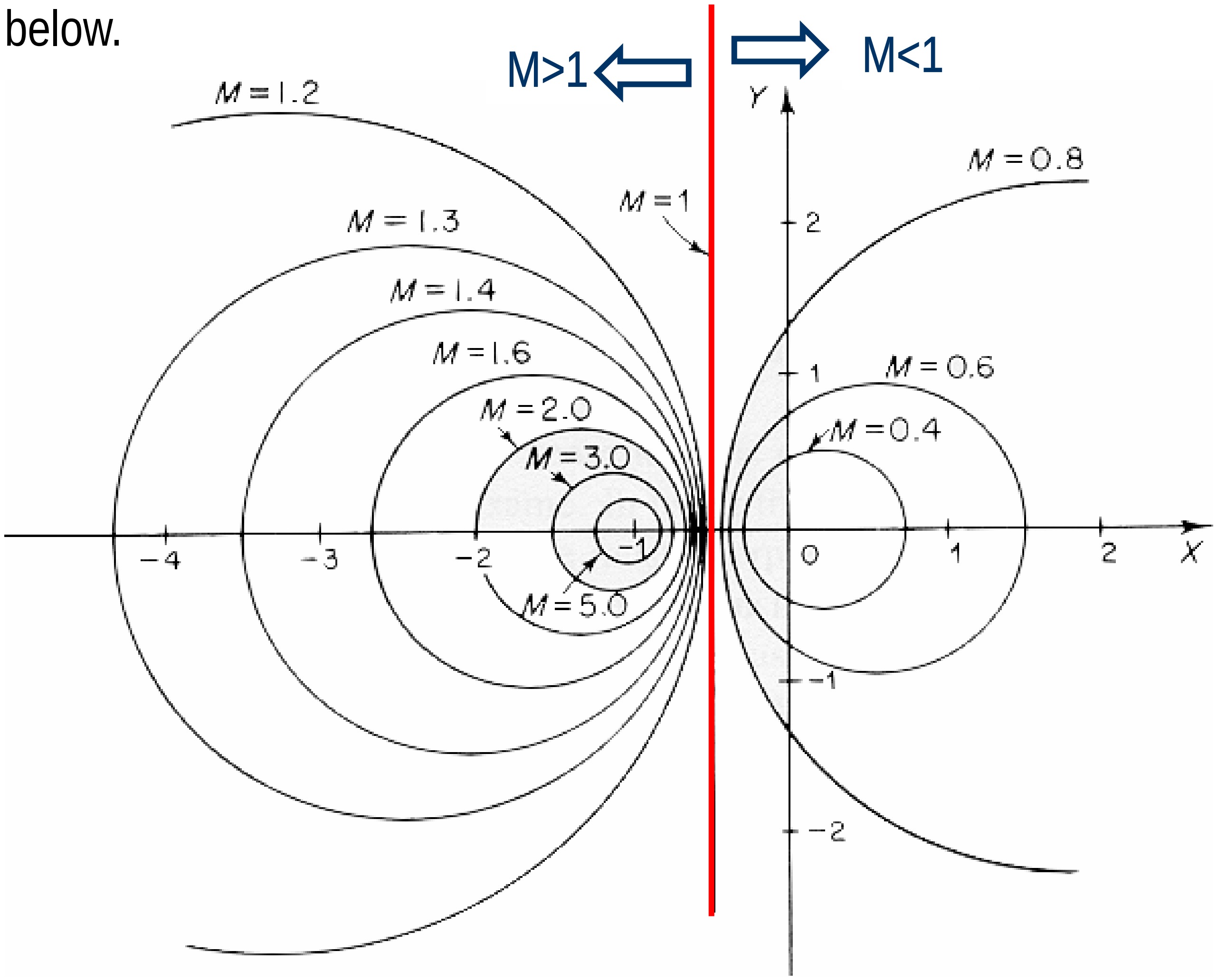
$$\left[X+\frac{M^2}{M^2-1}\right]^2+Y^2=\left(\frac{M}{M^2-1}\right)^2$$

circle on X-Y

Center location

radius

That is, all (X, Y) pair corresponding to a constant value of M for a circle on the complex plane. Therefore, we have the following (constant) M circles on the complex plane as shown below.



§**N circles** (constant phase of T)

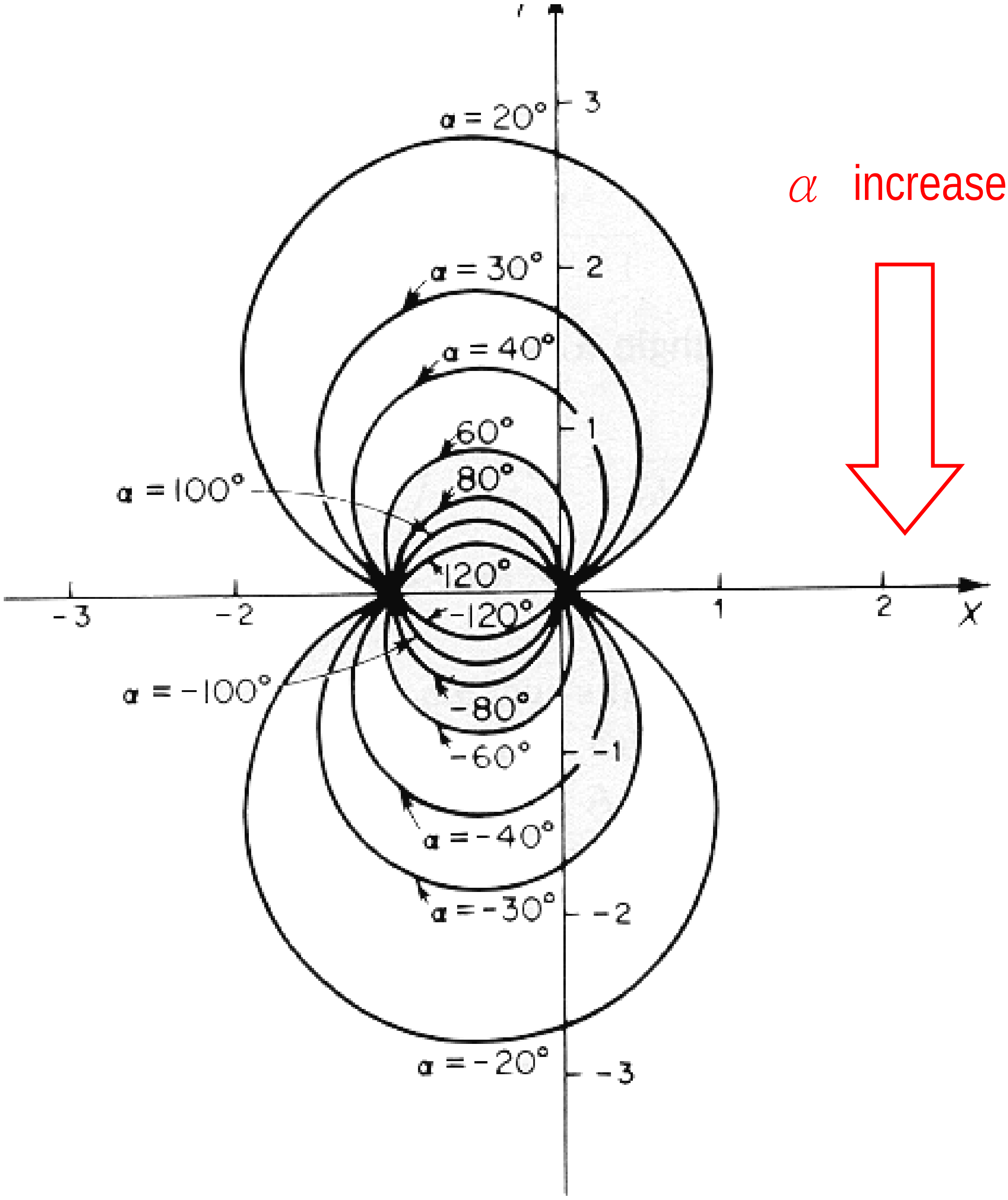
Similarly, it can be shown that the phase of $T(j\omega)$ be

$$\alpha \triangleq \angle T(j\omega) = \tan^{-1}\left[\frac{Y}{X}\right] - \tan^{-1}\left[\frac{Y}{1+X}\right]$$

Define $N \triangleq \tan\alpha = \tan\left(\angle T(j\omega)\right)$

It can be shown that all (X, Y) pair which corresponds to the same constant phase of T (i.e., constant N) forms a circle on the complex plane as shown below.

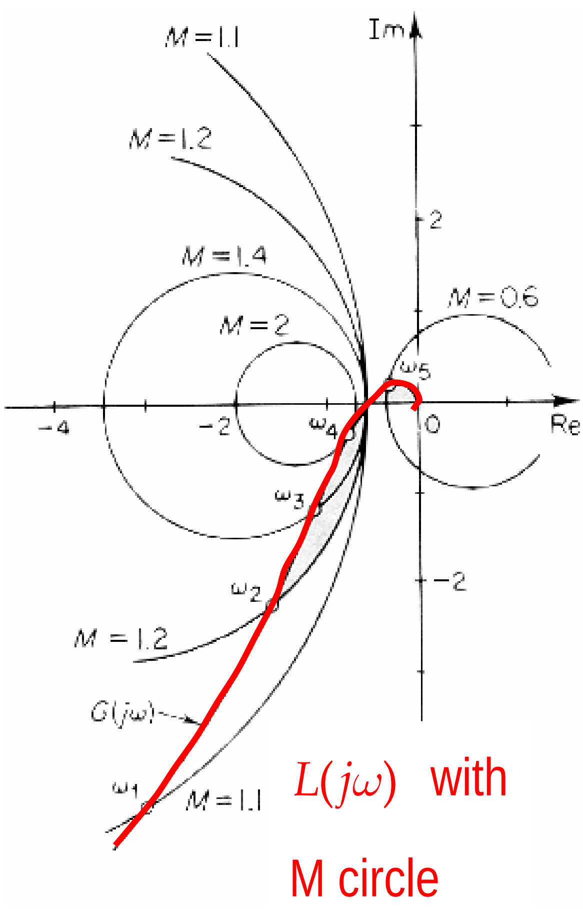
$$\left[X+\tfrac{1}{2}\right]^2+\left[Y-\tfrac{1}{2N}\right]^2=\tfrac{1}{4}+\left[\tfrac{1}{2N}\right]^2$$



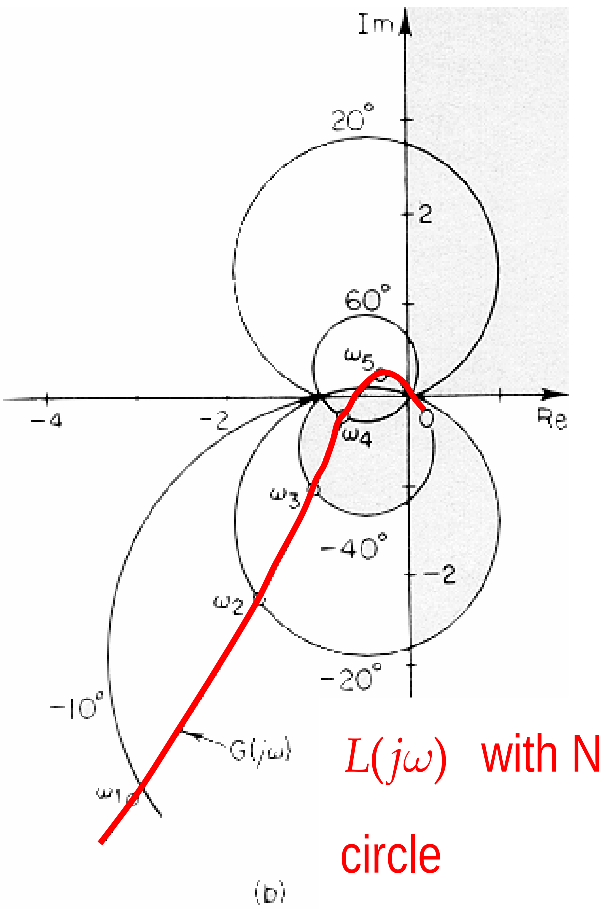
α increase

Constant N (phase angle α) circle

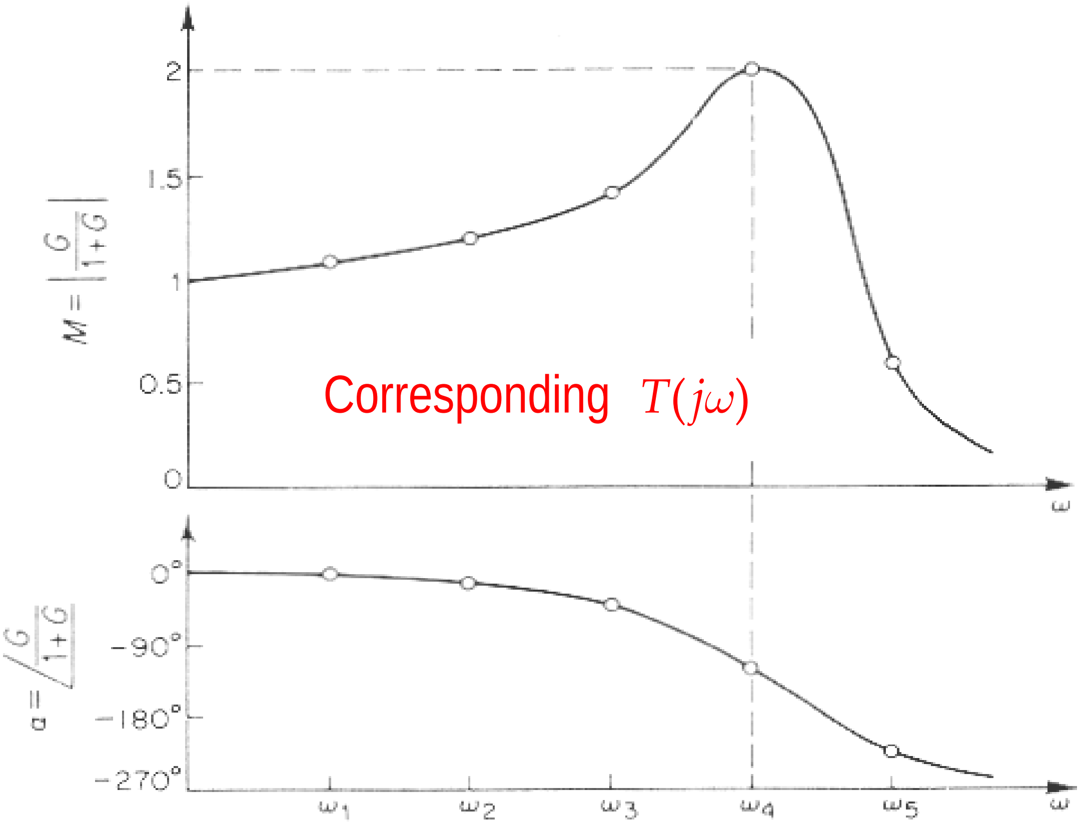
Example：Nyquist plot of $L(j\omega)$, and M-N circles of $T(j\omega)$



$L(j\omega)$ with M circle



$L(j\omega)$ with N circle



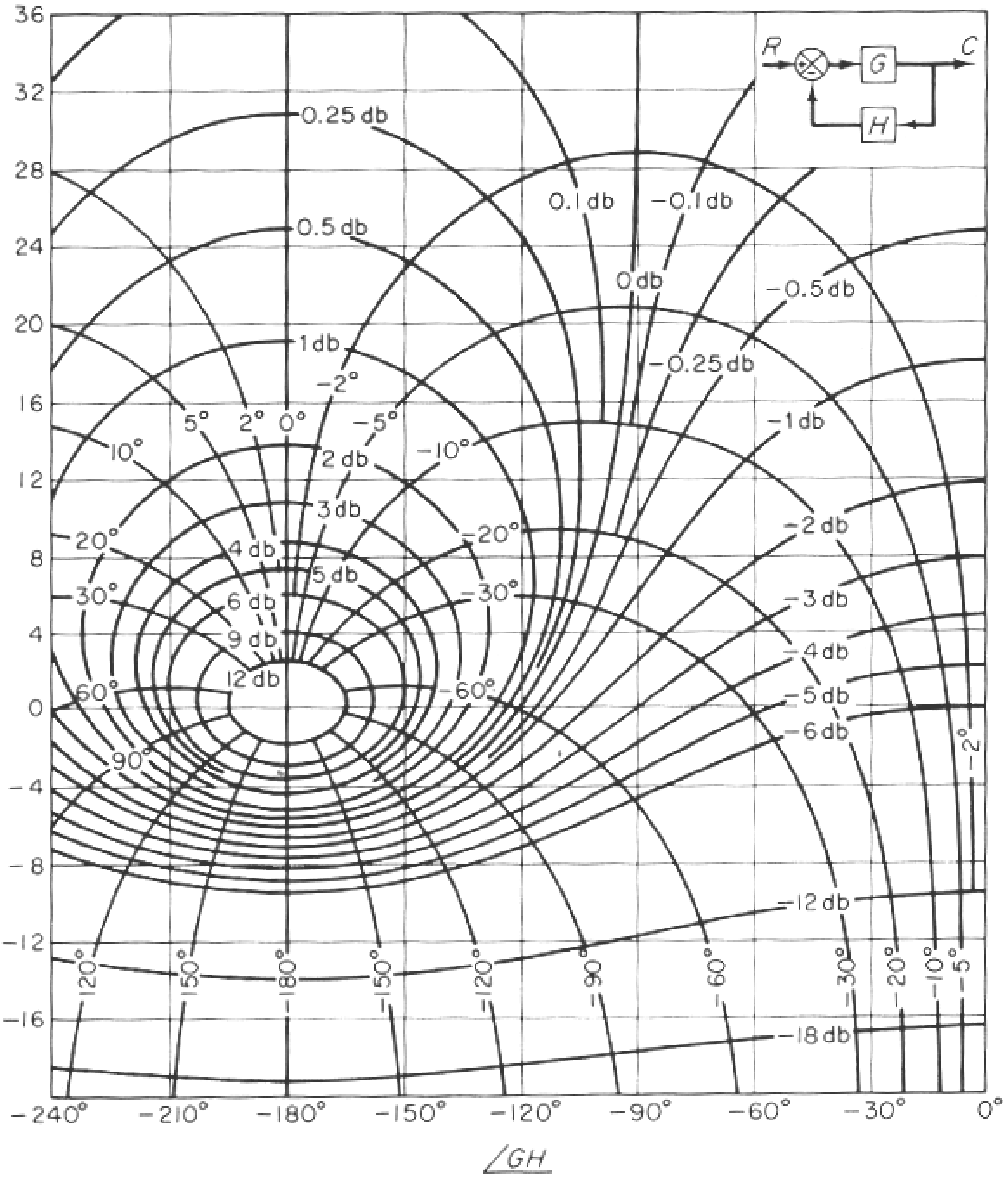
Corresponding $T(j\omega)$

§Nichols Chart

The Nyquist plot of $L(j\omega)$ can also be represented by its polar form using dB as magnitude and degree as phase.

$$L(j\omega)=|L|_{dB}\,e^{j\alpha}$$

All $L(j\omega)$ which corresponds to a constant $|T(j\omega)|$ can be draw as a locus of M circle on this plane as shown below

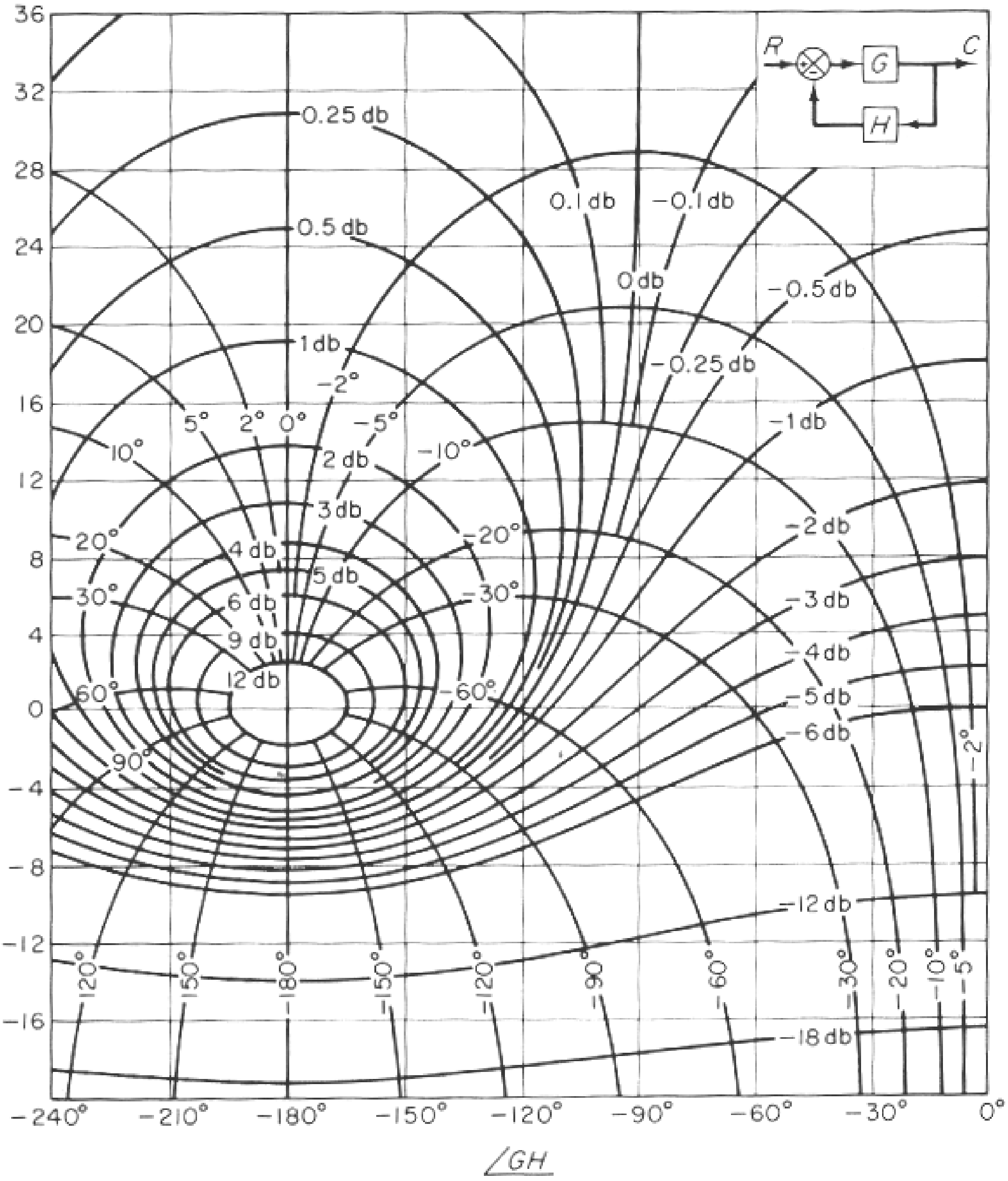


$|L|_{dB}$

$\angle L(\text{deg})$

M circles on Nichols chart

And ll $L(j\omega)$ which corresponds to a constant $\alpha(j\omega)$ can be draw as a locus of M circle on this plane as shown below

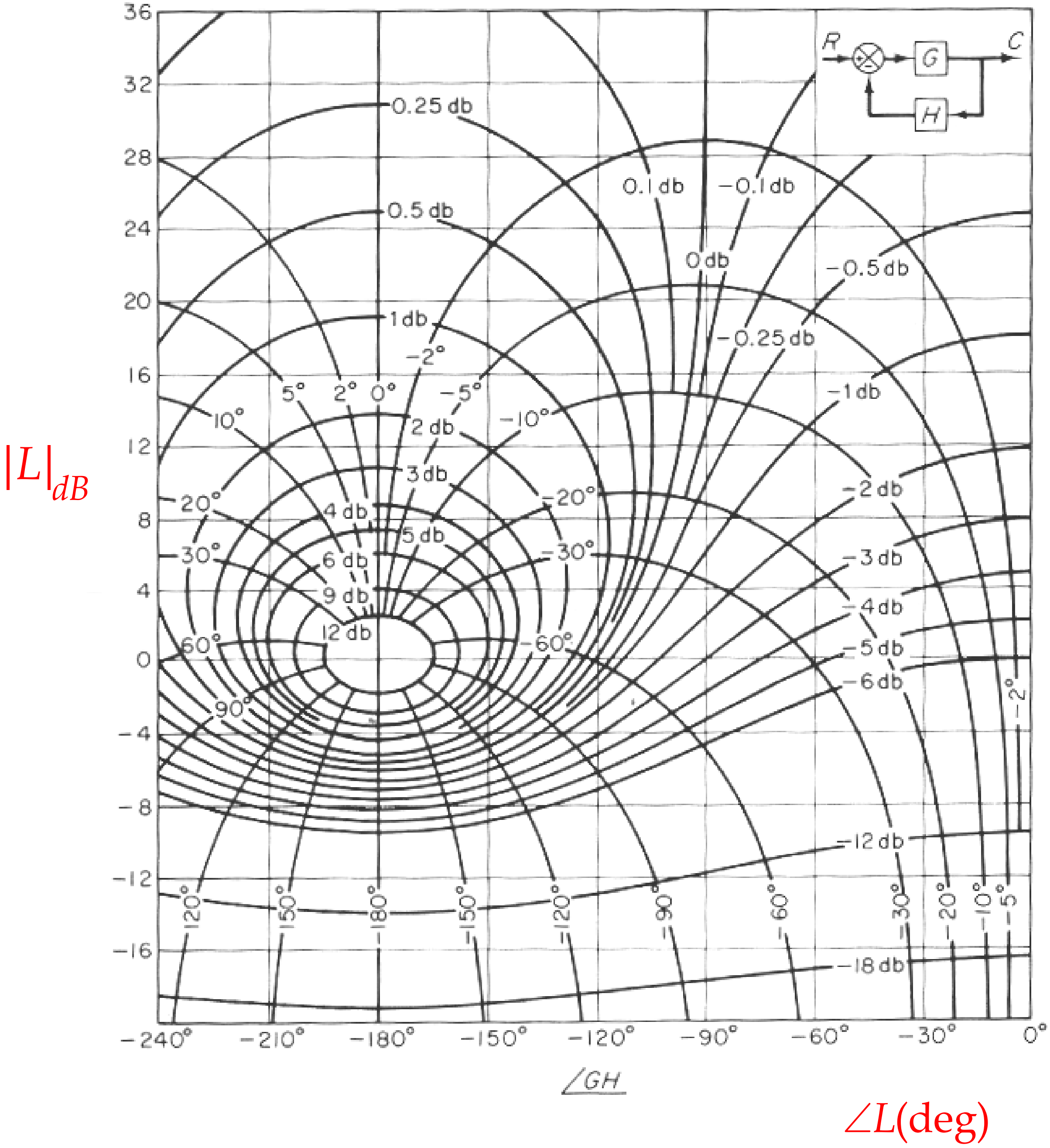


$|L|_{dB}$

$\angle L(^{O})$

N circles on Nichols chart

Combining the above two groups of M circles and N circles, we have the Nichols chart as shown below



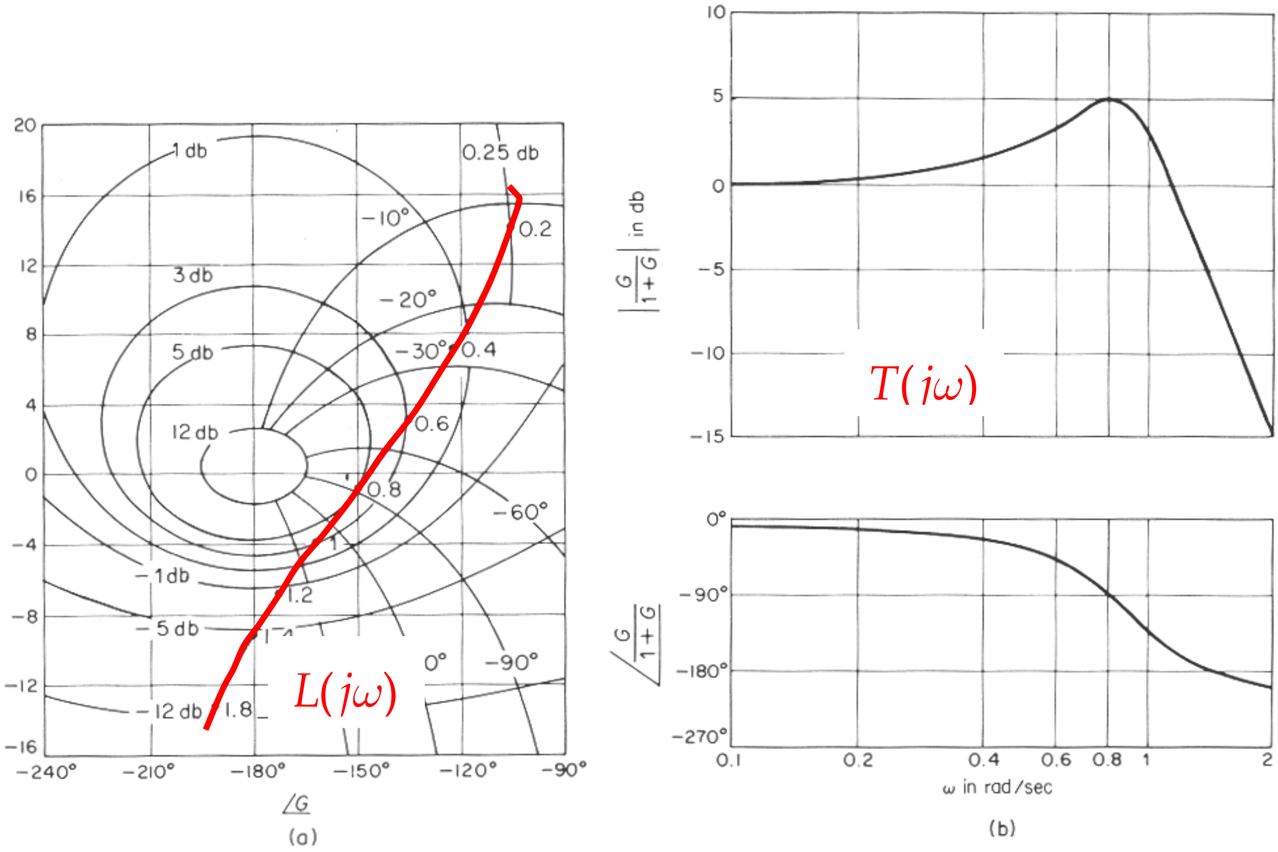
M circles and N circles on Nichols chart

Example 1

Given $L(j\omega)$ and a Nichols chart,

(1)Darw $L(j\omega)$ on Nichols chart frequency point by freq. point.

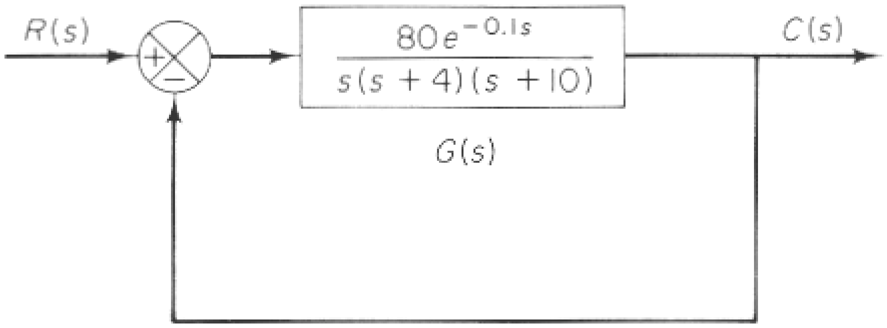
(2)Read Max $|T(j\omega)|$ = 12dB at ω =0.8rad/sec from the chart.

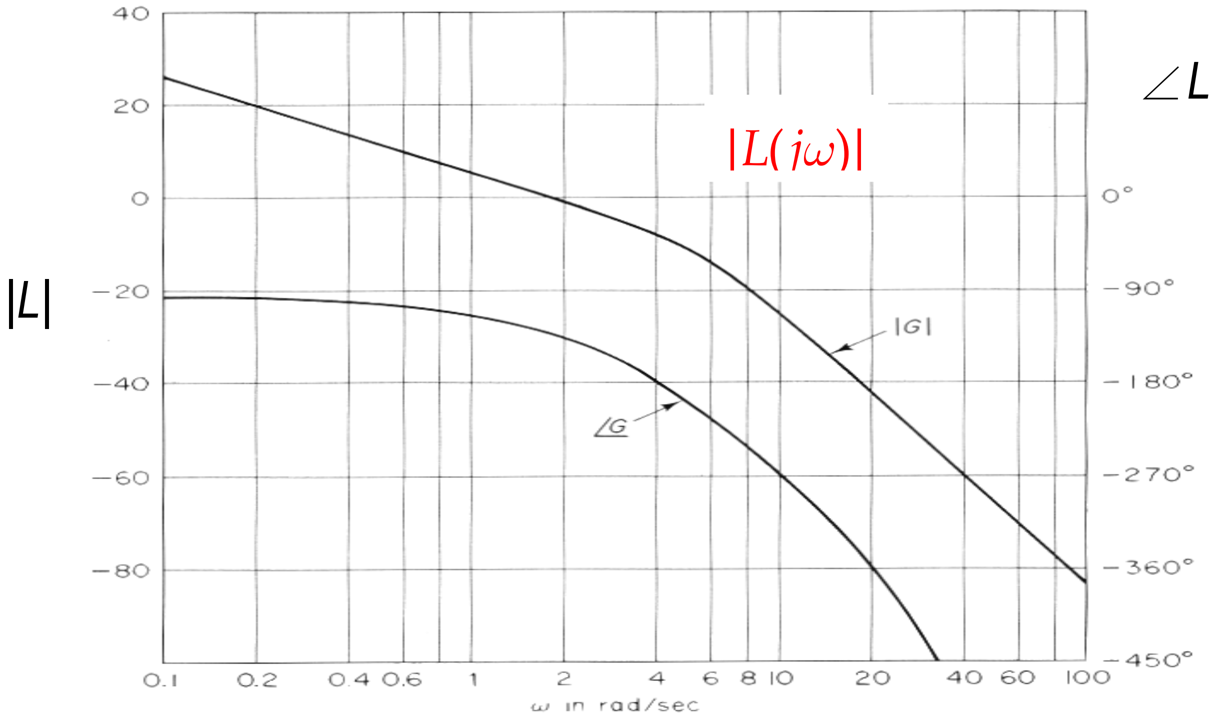


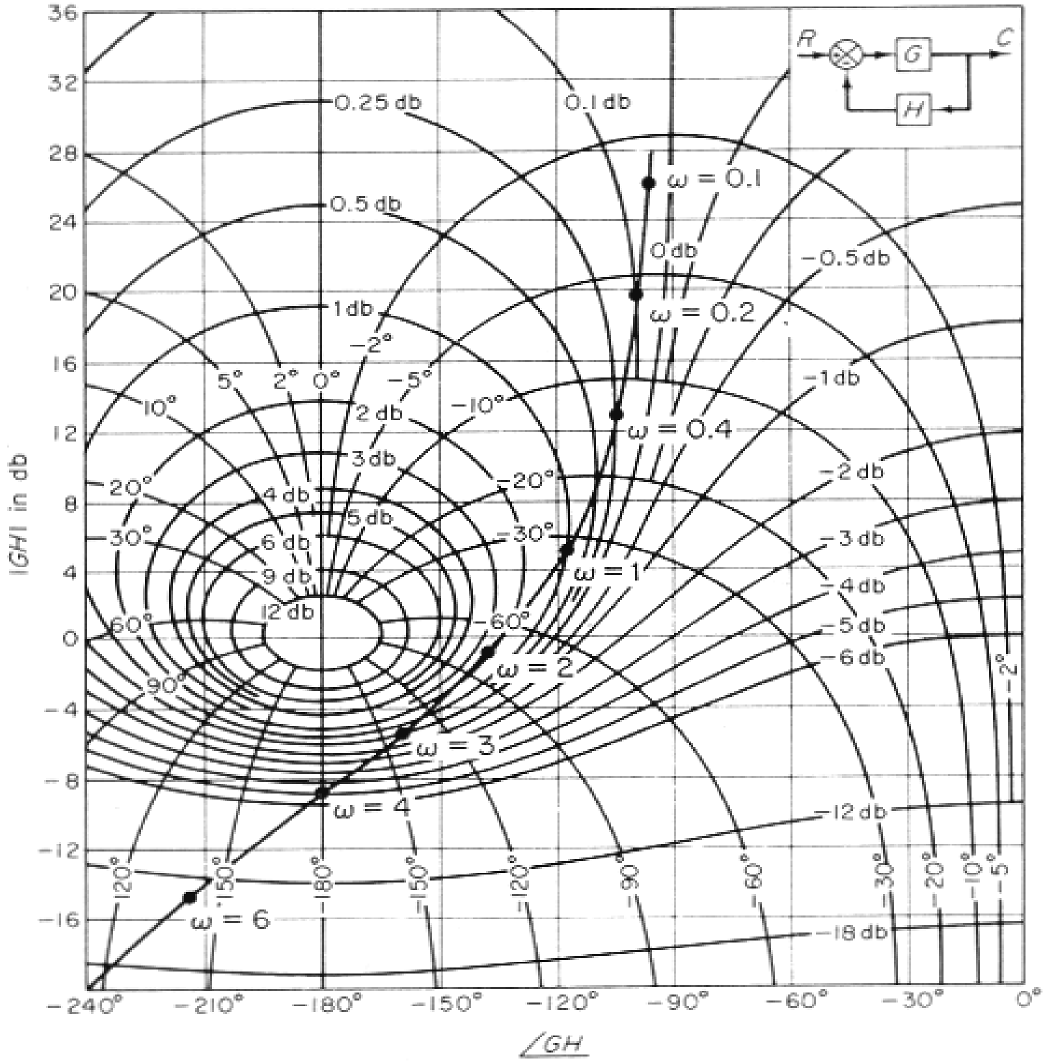
(a)

(b)

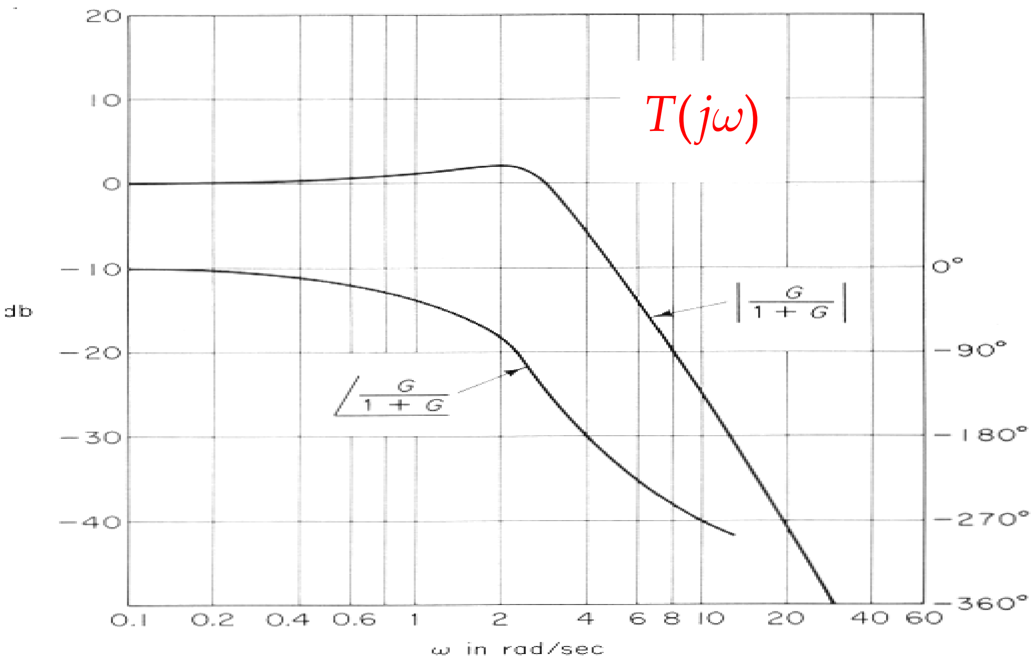
Example 2

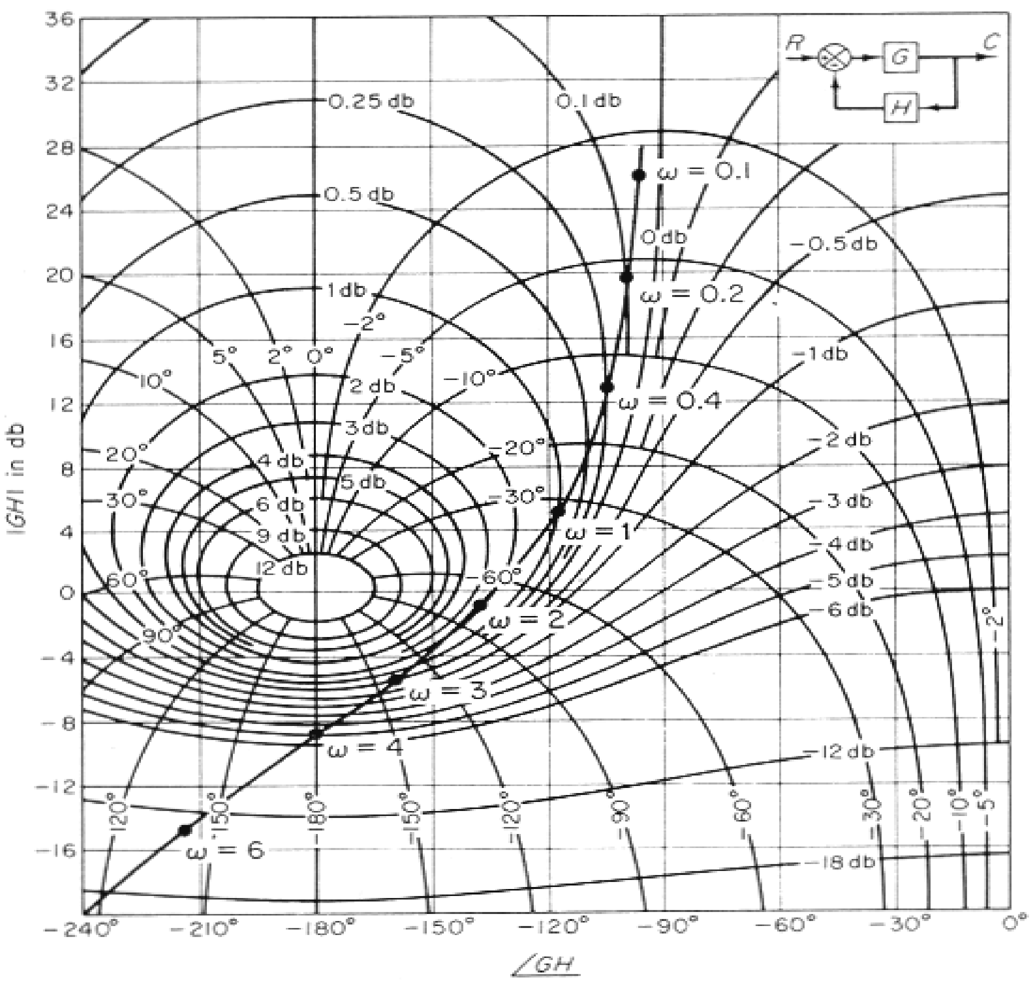



|L|
∠L
|L(jω)|
40
20
0
−20
−40
−60
−80
0°
−90°
−180°
−270°
−360°
−450°
|G|
∠G
0.1
0.2
0.4 0.6
1
2
4
6 8 10
20
40 60
100
ω in rad/sec


R
G
C
H
|GH| in db
∠GH
36
32
28
24
20
16
12
8
4
0
−4
−8
−12
−16
−240°
−210°
−180°
−150°
−120°
−90°
−60°
−30°
0°
0.25 db
0.1 db
0.5 db
0 db
−0.5 db
1 db
−1 db
2 db
3 db
4 db
5 db
6 db
9 db
12 db
−2 db
−3 db
−4 db
−5 db
−6 db
−12 db
−18 db
5°
2°
0°
−2°
−5°
−10°
10°
20°
−20°
30°
−30°
60°
−60°
90°
120°
150°
−180°
−150°
−120°
−90°
−60°
−30°
−20°
−10°
−5°
−2°
ω = 0.1
ω = 0.2
ω = 0.4
ω = 1
ω = 2
ω = 3
ω = 4
ω = 6

and the resultant close-loop system has the following Bode plot





§Performance and stability of uncertain plant

A. Design on Bode plot

$$G(j\omega)=G_O(j\omega)\Delta(j\omega) \quad \text{where}$$

$$|\Delta(j\omega)|\in\left[0.2,\ 3\right],\quad \angle\Delta(j\omega)\in\left[-20^O,\ 30^O\right]$$

Than

$$\begin{aligned} L(j\omega)&=G_O(j\omega)D(j\omega)\Delta(j\omega)\\ &=L_O(j\omega)\Delta(j\omega) \end{aligned}$$

Hence

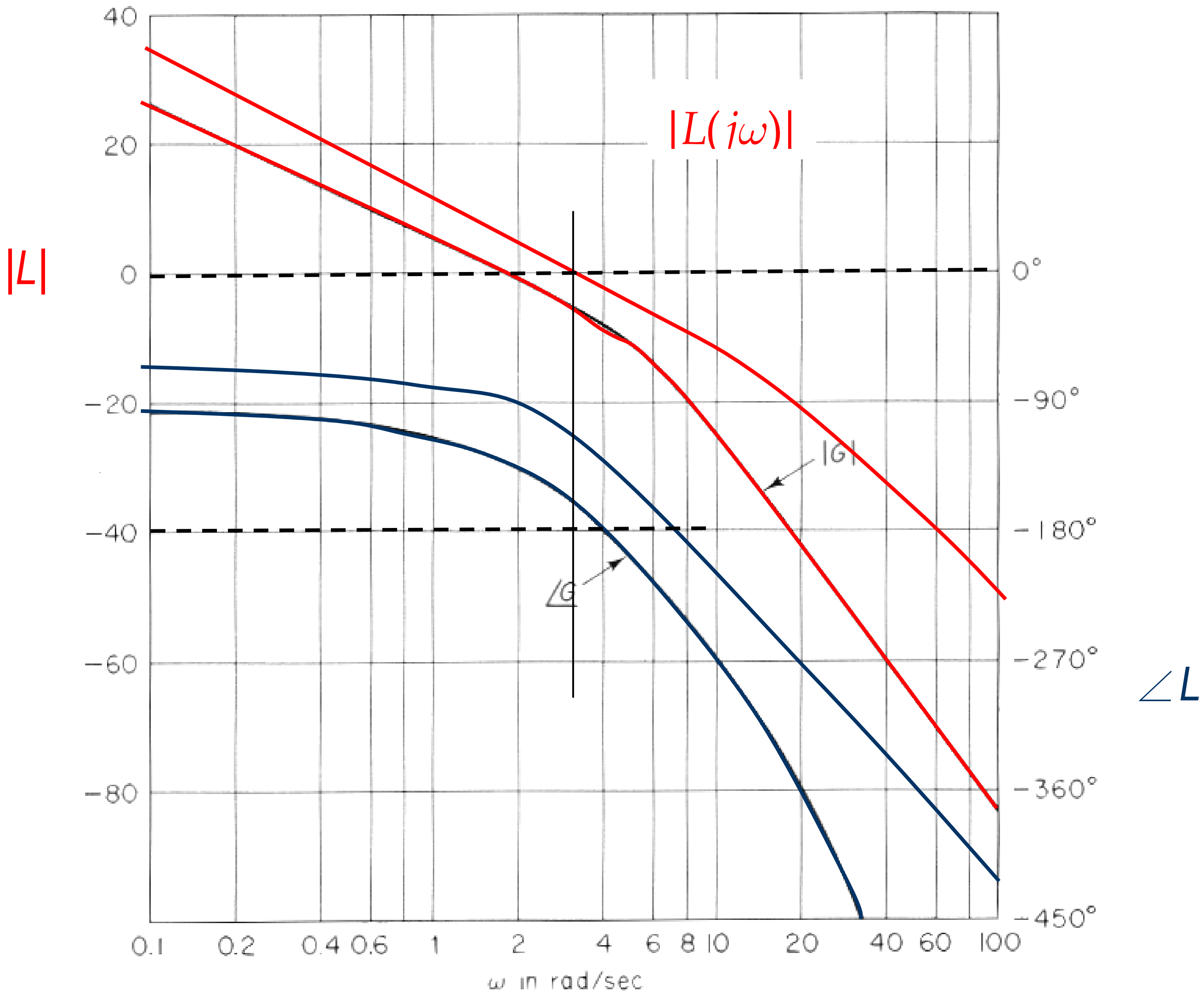
$$|G(j\omega)|_{dB}=|G_O(j\omega)|_{dB}\pm|\Delta(j\omega)|_{dB}$$

$$\angle G(j\omega)=\angle G_O(j\omega)\pm\angle\Delta(j\omega)$$

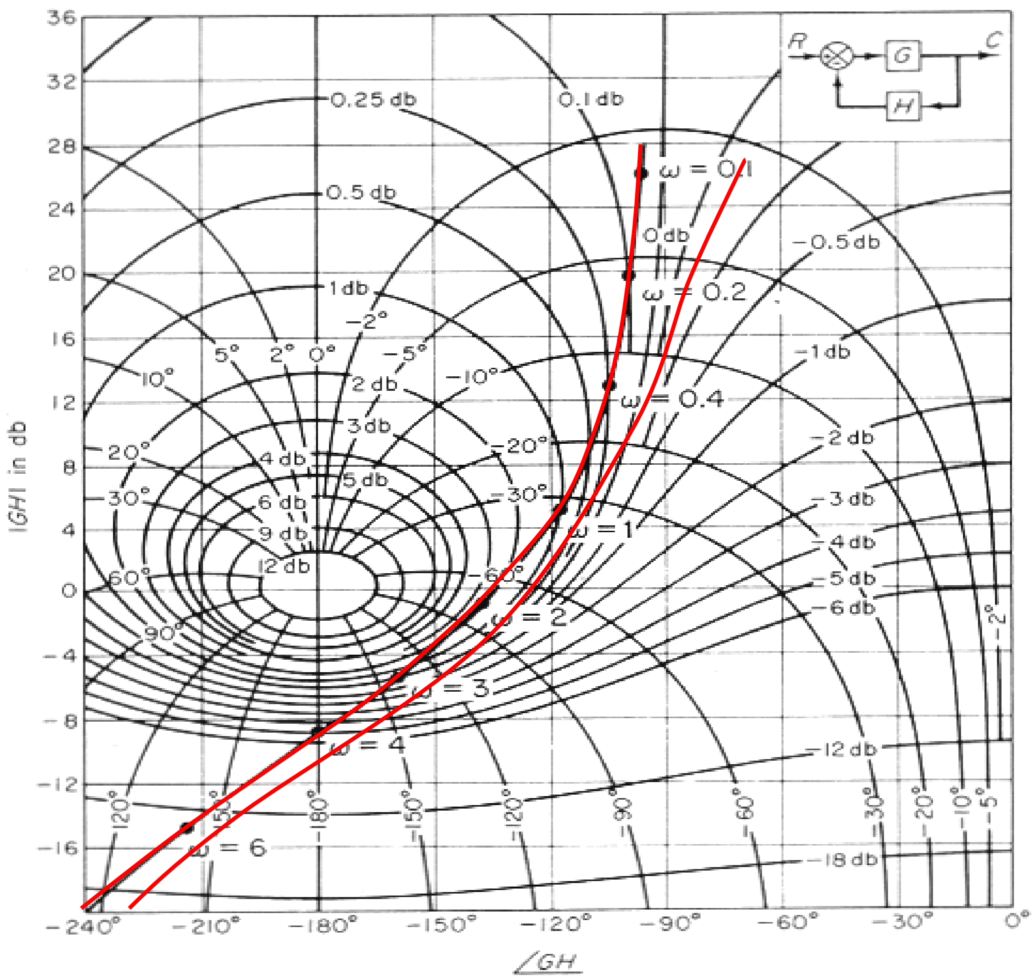
and

$$|L(j\omega)|_{dB}=|L_O(j\omega)|_{dB}\pm|\Delta(j\omega)|_{dB}$$

$$\angle L(j\omega)=\angle L_O(j\omega)\pm\angle\Delta(j\omega)$$



B. Design on Nichols chart



36
32
28
24
20
16
12
8
4
0
-4
-8
-12
-16
|GH| in db
-240°
-210°
-180°
-150°
-120°
-90°
-60°
-30°
0°
∠GH
R
G
H
C
0.25 db
0.1 db
0.5 db
0 db
-0.5 db
1 db
-1 db
2 db
-2 db
3 db
-3 db
4 db
-4 db
5 db
-5 db
6 db
-6 db
9 db
12 db
-12 db
-18 db
5°
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-2°
-5°
-10°
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-60°
90°
-90°
120°
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150°
-150°
-180°
-30°
-20°
-10°
-5°
-2°
ω = 0.1
ω = 0.2
ω = 0.4
ω = 1
ω = 2
ω = 3
ω = 4
ω = 6